Contents

Chapter 1

LOGICAL FOUNDATION OF MULTICRITERIA PREFERENCE AGGREGATION

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Abstract In this chapter, we would like to show Bernard Roy's contribution to modern computational logic. Therefore we first present his logical approach for multicriteria preference modelling. Here, decision aid is based upon a refined methodological construction, that provides the family of criteria with important logical properties giving access to the concordance principle used for aggregating preferential assertions from multiple semiotical points of view. In a second section, we introduce the semiotical foundation of the concordance principle and present a new formulation of the concordance principle with its associated necessary coherence axioms imposed on the family of criteria. This new methodological framework allows us, in a third part, to extend the classical concordance principle and its associated coherence axioms imposed on the family of criteria – first to potentially redundant criteria, – but also to missing individual evaluations and even partial performance tableaux.

Keywords: Multicriteria preference modelling, Electre decision aid methods, concordance principle

Foreword

Let me thank beforehand the editors for having invited me to contribute to this book in honour of Bernard Roy. When I obtained in summer 1975 a three years NATO fellowship in Operations Research, I got the opportunity to join, apart from several universities in the USA, two European OR laboratories. One was directed by H.-J. Zimmermann in Aachen and the other by Bernard Roy in Paris. Having made my under-graduate studies in Liège (Belgium), I knew well the nearby German city of Aachen and I decided therefore to preferably go to Paris and join Bernard Roy at the newly founded Université Paris-Dauphine. It is only later that I realized how important this innocent choice would be for my scientific career. Indeed, I joined the LAMSADE, Roy's OR laboratory, at a moment of great scientific activities. We may remember that 1975 is the birth year of EURO, the Federation of European OR Societies within IFORS and more specifically the birth year of the EURO Working Group on Multicriteria Decision Aid coordinated by Bernard Roy so that I became an active participant in the birth of the European School in Operations Research. One can better understand 25 years later that joining the LAMSADE at that precise moment made an ever lasting positive effect on me. May Bernard recognize in this contribution, a bit of the scientific enthusiasm he has communicated to all his collaborators. Indeed, I rarely met any other person of such arguing clarity when trying to match formal logical constructions with pragmatic operational problems which often, if not always, appear uncertain and fuzzy in nature. It is my ambition in this chapter to continue with this tradition.

R.Bisdorff, June 2001

1. Introduction

> ”Du point de vue de la connaissance, nous sommes capables de connaître une chose au moyen de son espèce et nous sommes incapables de la nommer si nous ne la connaissons pas; par conséquent, si nous émettons une *vox significativa*, c'est que nous avons une chose à l'esprit.” (*Umberto Eco, Kant et l'ornithorynque* [11, p.437])

In this chapter, we would like to show Bernard Roy's contribution to modern computational logic. Indeed, his original logical approach to preference modelling via the concordance principle may be seen as fruitful attempt for answering from a logical point of view cognitive questions such as: ‘”How do we know preferences” and ”What will be if a preferential situation is believed to be true”. Thus he has taken a somehow orthogonal position with respect to main philosophical and mathematical logic, where attention has been more and more concentrated on the direct relation between a statement and a state of the world. By concentrating his methodological work on this “knowledgability”, he has come to explore by what mental operations and semantic structures, a decision maker is capable of understanding what is the meaning of preferential situations and in particular of outranking ones. In this sense, he has shown us the way of how to naturally enrich classical truth-functional semantics with a semiotical foundation.

First we present the logical approach for multicriteria preference modelling as promoted by Bernard Roy. Here decision aid is based upon a refined methodological construction that provides the family of criteria with important logical properties giving access to the concordance principle used for aggregating preferential assertions from multiple semiotical points of view. Generally, these properties are discussed via representation theorems showing the kind of global preference models that it is possible to construct from a coherent family of criteria (see [10, 9, 12]). In this contribution we shall however concentrate on the logical foundation of the concordance principle as revealed by the semiotics of Roy's methodology.

In a second section, we will therefore introduce the semiotical foundation of the concordance principle and present a new formulation of it with its associated necessary coherence axioms imposed on the family of criteria. A main result, a priori a negative one, will be to make even more apparent the well known Achilles' heel of the concordance principle, i.e. the necessarily numerical (cardinal) assessment of the importance weights associated with the family of criteria.

But our methodological framework will allow us in a third part, and this was our main motivation for undertaking this research, to extend the classical concordance principle and its associated coherence axioms imposed on the family of criteria – first to potentially redundant criteria, – but also to missing individual evaluations and even partial performance tableaux. These extensions, we hope, should help making the concordance principle and thereby the logical approach to preference modelling as promoted by B. Roy more convincing for applications in decision aid.

2. How to tell that a preferential assertion is true?

In this section we present the constructive approach to multicriteria preference modelling proposed by Bernard Roy (see [17, 18]). In order to describe the preferences a decision maker might express concerning a given set of decision actions, we consider essentially the multiple pragmatic consequences they involve. On the basis of these consequences we introduce a family of criterion functions for partial truth assessment of desired preferential assertions, namely outranking situations. Aggregating multiple partial truth assessments of these outranking assertions will be achieved via the concordance principle. To give adequate results, this concordance principle imposes necessary coherence properties on the underlying family of criteria.

2.1. Describing decision action's consequences from multiple points of view

We assume at this place that in a given decision problem, a set A of potential decision actions has been defined and recognized by the actual decision maker. Our main interest goes now to describing the decision maker's preferences concerning these decision actions. In our discussion we restrain our interest to preferences expressed as pairwise *outranking*, i.e. "*to be at least as good as*" situations on A.

In a *constructive pragmatic* way, Roy states that "*every effect or attribute characterizing a given decision action* $a \in A$ *which could interfere with the op-*

erational goals or the ethic position of the decision-maker as a primary element to elaborate, justify or transform his/her preferences is called a consequence *of* a" (Roy [17]).

Definition 1. To speak of all possible such consequences of the decision actions A before any formal decision-aid activity has been going on, Roy introduces the concept of *cloud of consequences* denoted $\nu(A)$.

Modelling this cloud of consequences consists first in identifying *elementary consequences*, i.e. semantically well recognized effects or attributes with well defined and observable states describing the consequences that would occur if a potential decision action a is going to be executed.

A strong pragmatic commitment is taken here by Roy with respect to what kind of consequences will contain the formal model of the cloud of consequences. Indeed, no vague impressions, intuitions or beliefs are supposed to be taken into account.

Being principally interested in capturing an adequate semiotical reference of preferential assertions, Roy restricts his attention to such elementary consequences that support a preference dimension.

Definition 2. A *preference dimension c* is an elementary consequence such that the set of its possible states may be organized as a *preference scale* E_c, i.e. a total order $(E_c, \leq)$ with the following property : considering two ideal decision actions a and b which may be compared exactly with the help of two states e and e' of E_c, then a and b are considered indifferent iff $e = e'$, whereas a is considered to be preferred to b iff $e > e'$.

The complete set of preference dimensions on which all elementary consequences of all the decision actions may be completely and operationally described is called the *consequence spectra* of the decision actions and denoted $\Gamma(A)$.

Two important constructive implications may be outlined at this point: – first, the cloud of consequences is split into a generally small number of elementary consequences, well identified and recognized as preference dimensions by the decision-maker; – secondly, each such elementary consequence gives support to some kind of *independent preference assessment* on the set A through a "*toute chose pareille par ailleurs*" reasoning principle.

Summarizing, we notice that the elaboration of a consequence spectra $\Gamma(A)$ follows precise methodological requirements (see Roy [17, p.220]) that are:

- an *intelligibility principle*: Its components must gather as directly as possible all imaginable consequences such that the decision maker is able to understand them with respect to each of the m preference dimensions.

- an *universality principle*: The components must ideally cover all preference dimensions that reflect fundamental and unanimous outranking judgments concerning the set of all decision actions in A.

2.2. Partial truth assessment of outranking assertions

Following the exhaustive formal description $\Gamma(a)$ of the individual multiple consequences of a decision action $a \in A$, Roy now introduces the concept of *criterion function.*

Definition 3. A criterion function $g : A \rightarrow \mathbb{R}$ is a real-valued function, defined on the set of potential decision actions A, which captures operationally the preferential description of a determined part $\Gamma_g(A)$ of the consequence spectra, called the support of g. Such a criterion function verifies therefore the following operational conditions:

- The number $g(a)$ is defined iff the sub-cloud of consequences $\nu_g(a)$ taken into account by the criterion function g is effectively evaluated in a given sub-spectra $\Gamma_g(a)$.
- The decision-maker recognizes the existence of a *significant* preference dimension with respect to which two decision actions a and b may be compared relatively to the only consequences covered by $\Gamma_g(a)$ and (s)he accepts to model this comparison as follows[1]:

$$g(a) + d \geq g(b) \;\;\Leftrightarrow\;\; a\mathrm{S}_g b,$$

  where $d \in \mathbb{R}_+$ represents a possible indifference threshold and S_g stands for the semiotical restriction of an outranking relation S to the sub-spectra $\Gamma_g(a)$ covered by criteria g.

Roy is using the concept of *'criterion'* in the sense of a formal basis, a model for supporting preferential judgments. For any two decision actions a and b, a given criterion function g allows to *warrant truth or falseness* of the global outranking assertion '$a\,\mathrm{S}\,b$' with respect to a recognized part $\Gamma_g(a)$ of the consequence spectra covered by criterion g.

Definition 4. A *family F of criteria* constitutes a finite set of criterion functions that cover the whole consequence spectra $\Gamma(a),\ \forall a \in A$. Evaluating all decision actions on such a family of criteria results in a *performance tableau* $T = (A, F)$, i.e. a two-way table representing $g(a)$ for each decision action $a \in A$ on each criterion function $g \in F$.

[1]Neglecting in this definition a possible indifference threshold, Roy uses normally a single implication. But we prefer to work with a double implication as it allows to capture at the same time the semantics of the negated assertion.

Here, the term *'family'* refers to the fact that the considered set of criterion functions supports exhaustively the pragmatic preferences of the decision-maker. More generally, we notice in Definition 3 that the universal assertion '$a \, \mathrm{S} \, b$' is *truth or falseness warranted from multiple points of view* depending on the decomposition of its cloud of consequences into separated preference dimensions.

2.3. The concordance principle

A given performance tableau $T = (A, F)$, involving a set A of decision actions and a family F of criteria, allows a partial truth assessment of pairwise outranking situations along all individual criterion. It is the refined constructive methodology that gives the decision maker the ability to clearly acknowledge such partial outranking situations on behalf of the performances.

In order to aggregate now these partial outranking assertions, we need to consider the significance, each individual criterion takes in the eyes of the decision maker, for assessing the truth of the corresponding universal outranking situation.

Definition 5. Let $T = (A, F)$ be a given performance tableau and let $k_g \in \mathbb{Q}_+$, measure numerically the significance, criterion $g \in F$ takes in the eye of the decision maker with resect to the truth assessment of the universal outranking situation. Let k_F denote the universal closure of the significance weights over the whole family of criteria, i.e. $k_F = \sum_{g \in F}(k_g)$. We denote F^+, the subset of criteria that clearly support the truthfulness of a given outranking assertion and we define the *credibility* $r(a \, \mathrm{S} \, b)$ of the universal outranking assertion '$a \, \mathrm{S} \, b$' as follows:

$$r(a \, \mathrm{S} \, b) = \sum_{g \in F^+} \left(\frac{k_g}{k_F}\right).$$

If $r(a \, \mathrm{S}_g \, a') \geq \frac{1}{2}$, '$a \, \mathrm{S} \, b$' is considered to be *more or less true.*

The credibility of the universal outranking situation is computed as the sum of the relative weight of the subset F^+ of criteria confirming truthfulness of this assertion. If a majority of criteria is *concordant* about supporting the given outranking situation, it may be affirmed to be more or less *true* depending on the effective majority it obtained. Following Roy, this *logical concordance principle* may be interpreted as a *voting mechanism* in favour of the truth concerning a given outranking assertion, each criterion $g \in F$ participating in the voting with a number of voters equivalent to the amount k_g of knowledge concerning the truth assessment it supports.

2.4. Necessary coherence of the family of criteria

The performance tableau, representing a synthetic description of the consequence spectra, thus appears as an essential, but also very difficult step in a practical decision aid problem. Indeed, as pointed out by Roy (see [17] p. 310), besides a cognitive problem of acceptance of the criteria by the decision-maker, there are the following logical requirements to respect when constructing a family of criteria:

- *Exhaustivity* of the family of criteria: All individual consequences, out of $\Gamma(a)$ and $\Gamma(b)$ for two decision actions a and b and of relevance for their mutual comparison in terms of preference or indifference, have to be taken into account. This requirement takes its origin in the universal closure of the relative significance weights of the criteria over the whole family of criteria used in Definition 5.

- *Cohesion* between local preferences, modelled at the level of the individual criterion, and global preferences modelled by the whole family of criteria: Global preference judgments must coherently reflect themselves when transposed into individual criteria based preferences. The decision maker recognizes a clear universal outranking situation $'a \, \mathrm{S} \, b'$ whenever the performance level of action a is significantly better than that of action b on one of the criteria of positive significance, performance levels of these actions staying the same on each of the remaining criteria. This requirement guarantees separability of the individual preference dimensions which in term allows the additive computation of the credibility degrees.

- *Non-redundancy* of the criteria: The family is *minimal* with respect to both preceding properties. Again the importance, that each criterion will take in the truth assessment of an outranking situation via the concordance principle, is coherently measured only if no redundant consequences are taken into account.

Definition 6. A family of criteria, verifying the exhaustivity, the cohesion and the minimality requirement is called a *coherent* family.

Before discussing more thoroughly in Section 3 these coherence properties from a semiotical point of view, let us first turn our attention to the logical denotation, the credibility calculus as resulting from the concordance principle, transfers to outranking assertions.

2.5. Truth assessment by balancing reasons

> ”The rule for the combination of independent concurrent arguments takes a very simple form when expressed in terms of the intensity of belief ... It is this: Take

> the sum of all the feelings of belief which would be produced separately by all the arguments *pro*, subtract from that the similar sum for arguments *con*, and the remainder is the feeling of belief which ought to have the whole. This is a proceeding which men often resort to, under the name of *balancing reasons*.", (C.S. Peirce, *The probability of induction*, [16]).

Inspired by Peirce's proceeding of balancing reasons as quoted above, we may reformulate the concordance principle in the following way:

Definition 7. Let A be a set of decision actions evaluated on a coherent family of criteria. Let S denote an outranking relation defined on A. For all $a, b \in A$, let F^{+} denote the subset of criteria in favour of the universal assertion '$a \,\mathrm{S}\, b$' and $F^{-} = F - F^{+}$ the complementary subset in F. We define the credibility $r'(a \,\mathrm{S}\, b)$ of assertion $'a \,\mathrm{S}\, b'$ as follows:

$$r'(a \,\mathrm{S}\, b) = \sum_{g \in F+} \left(\frac{k_g}{k_F}\right) - \sum_{g \in F^-} \left(\frac{k_g}{k_F}\right)$$

Following this definition, the degree of credibility of an outranking assertion implements a rational function on $A \times A$ varying between -1 and 1. If $r'(a \,\mathrm{S}\, b) = 1$ there is unanimity in favour of '$a \,\mathrm{S}\, b$' and if $r'(a \,\mathrm{S}\, b) = -1$ there is unanimity in disfavour of it. If $r'(a \,\mathrm{S}\, b) = 0$ both the reasons in favour and those in disfavour balance each other and there appears no clear denotational result.

Definitions 5 and 7 are linked through the following proposition.

Proposition 1. *Let* $r : A \times A \to [0, 1]$ *and* $r' : A \times A \to [-1, 1]$ *represent the computation of the degrees of credibility of the outranking relation* S *on* A *following Definition 5 respectively Definition 7. Then the following relation holds between* r *and* r'*:*

$$r' = 2r - 1 \tag{1.1}$$

Proof. Equation 1.1 results immediately from the following development:

$$\sum_{g \in F^-} \left(\frac{k_g}{k_F}\right) = 1 - \sum_{g \in F+} \left(\frac{k_g}{k_F}\right)$$

□

Proposition 1 evidently relies again upon the three properties of the coherent family of criteria and it has an interesting logical corollary.

Corollary 1. *Let* A *be a set of decision actions evaluated on a given coherent family of criteria* F *and let* $r(a \,\mathrm{S}\, b), \forall a, b \in A$ *denote the degree of credibility of a pairwise outranking situation computed following Definition 5.*

- *if* $r(a \,\mathrm{S}\, b) \geq \frac{1}{2}$*, then '*$a \,\mathrm{S}\, b$*' is considered to be more or less* true,

- *if* $r(a\,\mathrm{S}\,b) \leq \frac{1}{2}$ *then '*$a\,\mathrm{S}\,b$*' is considered to be more or less* false *and finally,*
- *if* $r(a\,\mathrm{S}\,b) = \frac{1}{2}$ *then '*$a\,\mathrm{S}\,b$*' is considered to be logically* undetermined.

Proof. The linear transformation of Equation 1.1, representing an order isomorphism between credibility degrees r and r', gives a faithful transformation from the truth denotation of Definition 7 to the truth denotation of Definition 5, in the sense that

$$r(p) \geq \frac{1}{2} \Leftrightarrow r'(p) \geq 0.$$

In its truth denotation, the concordance principle is therefore isomorphic to the balancing reasons proceeding. □

It is important to notice that the refutation of an outranking situation '$a\,\mathrm{S}\,b$' in case we observe its credibility to be below $\frac{1}{2}$, does not necessarily induce that the converse outranking situation, i.e. '$b\,\mathrm{S}\,a$' should be automatically affirmed. On the contrary, even when we may observe complete preorders on A on every single criterion, the concordance principle commonly generates universal outranking relations on A that give no complete preorders, even no partial preorders anymore, as no global transitivity is formally implied by Definition 5.

The split truth versus falseness denotation, installed by the concordance principle appears as a powerful natural fuzzification of Boolean Logic (see Bisdorff [6]). Indeed, the algebraic framework of the credibility calculus, coupled to its split logical denotation, allows us to solve selection, ranking and clustering problems (see Bisdorff [3, 5, 8]) directly on the base of a more or less credible pairwise outranking relation without using intermediate cut techniques as is usual in the classic Electre methods (see Roy & Bouyssou [18]).

As mentioned earlier, the concordance principle requires the assessment of cardinal significance weights for all criteria. This requirement represents a well known weak point when it comes to practical decision aid. Numerous theoretical and empirical efforts have therefore been devoted to develop adequate methodologies for helping define these weight coefficients for different preference aggregation methods (see Roy & Mousseau [19, 15]), but few have considered the essentially semiotical nature of the credibility calculus installed trough the balancing reasons proceeding.

3. Semiotical foundation of the concordance principle

In this section we will therefore explore in depth the relationship between a given family of criteria and its semiotical interpretation in terms of the underlying cloud of consequences. Our approach closely follows the classical measure-theoretical axiomatization of probability theory. Where random events support the probability measure, we use semiotical interpretations to support the credibility calculus. The *amount of truth assessment knowledge* carried by the family of criteria is thus supported by the denotational semantics of the family of criteria with respect to the given consequence spectra. In this way we axiomatize the concordance principle in a measure-theoretical way and a new version of the coherence axioms of the family of criteria is presented.

3.1. Credibility versus state of belief

Indeed, following the Peircian discrimination between degree of credibility and state of belief ([16, p.175]) gives us a broader approach to the problem of evaluating the credibility, a decision-maker should have in the proposition that a certain action a outranks another action b on behalf of a given performance tableau. Indeed, Peirce states that : " *to express the proper state of our belief, not one number but two are requisite, the first depending on the inferred* [credibility], *the second on the amount of knowledge on which that* [credibility] *is based* "[2]

When the exhaustivity of the family of criteria is given, a single degree of credibility is solely sufficient for expressing our belief in a given outranking assertion. But when such exhaustivity is not given, the second number, the actual amount of knowledge used to assess the truthfulness of this assertion becomes important.

What Peirce means in general here, refers to the fact that solely considering a relative credibility degree or ratio, is necessarily restricted to the condition that an universal, i.e. constant amount of truth assessment knowledge underlies all arguments.

To illustrate the point, we may indeed consider that the family of criteria represents a global voting assembly with a certain number of individual voters, each one representing one of the given criteria. This assembly is split into sub-assemblies, each representing the preference dimension modelled by one of the possible criteria.

Following this metaphor, the three basic requirements, a coherent family of criteria has to meet in order to comply to the concordance principle, may be understood as follows: – first, concerning exhaustivity, we have to assume that the union of all sub-assemblies completely returns the global assembly. No significant voters concerning the truth assessment of a given assertion are missing in the global assembly; – secondly, to guarantee the necessary separability condition, all voters must participate in at most one sub-assembly; – finally, the minimality condition imposes that each partial point of view must be represented by at most one sub-assembly.

[2] We have added the credibility term ([16, p. 179]).

Under these conditions, we may compute the credibility of a given outranking situation simply by dividing the sum of positive votes collected in each sub-assembly by the sum of voters of the global assembly. The concordance principle thus appears as a *weighted average of truth assessments from multiple points of view*.

More fundamentally, the concept of *significance*, understood as the *amount of truth assessment knowledge* modelled by a single criterion, a coalition of criteria, or even by the exhaustive family of criteria appears to be of utmost importance.

We now introduce an explicit axiomatization for measuring this knowledge in the context of the multicriteria preference aggregation via the concordance principle.

3.2. Basic semiotics for a logical credibility calculus

Let A be a set of potential decision actions upon which a decision maker M wishes to describe his outranking preferences $S \subseteq A \times A$. Let $T(A, F)$ represent the performance tableau elaborated in a decision aid process. In order to simplify our presentation we may assume that each criterion-function $g \in F$ is modelling a different single elementary preference dimension.

Let '$a \,\mathrm{S}\, b$' be an affirmative outranking assertion. What we have to axiomatize, is the precise measurement of the amount of truth assessment knowledge each preference dimension, identified in the consequence spectra $\Gamma(A)$, brings in.

Definition 8. We call *referential evaluation* of '$a \,\mathrm{S}\, b$' with respect to the subset $J \subset F$ of criteria, the *interpretation* of the pair $(g_J(a), g_J(b))$ of performances of decision actions $a, b \in A$ on the subset J of criteria in terms of the *practical significance* of the concerned subset of elementary consequences, for warranting truth or falseness of '$a \,\mathrm{S}\, b$'. We call *semiotical reference* and denote $R(J)$, the result of a referential evaluation restricted to a subset J of criteria.

In accordance with the universality principle of the construction of the consequence spectra $\Gamma(A)$ (see Section 2.1), we assume in the sequel that a referential evaluation remains *universally constant* for any given subset J of criteria over all possible pairs of decision actions [3].

Definition 8 installs the cognitive process of interpreting criterial performances in terms of *pragmatic consequences.* In this way, we introduce into the decision aid model the pragmatic *goal* of the decision maker as well as his(her) *subjective value system*, i.e. his(her) proper subjective preference judgments.

[3]The universality of the semiotical reference over all possible pairs of decision actions, as assumed in Definition 8, is a highly problematic assumption from a cognitive point of view, but due to space limitations, we do not discuss this issue within this contribution.

Definition 9. We call *reference family* $\mathcal{R}_F$, the set of all semiotical references $R(J)$ for $J \subseteq F$ associated with a referential evaluation of an outranking relation through the family F of criteria. $R(J)$ is called an *elementary reference* if J is confined to a single criterion-function, whereas we call *empty reference*, a reference $R(J)$ such that its significance is zero. Otherwise $R(J)$ is called a *composed reference*. We call *exhaustive reference*, a composed reference $R(J)$ such that the significance of the set of criteria J covers the whole consequence cloud.

It is important to notice that a semiotical reference in our sense is different from the material states of the consequences we actually observe in the performance tableau via the corresponding criterion-functions. Here we are interested essentially in the *significance* of these states, i.e. the semiotics[4] of the relational formula '$a\,\mathrm{S}\,b$' in fact supported by the pairs $(g_J(a), g_J(b))$ of evaluations. These formulas are seen, in the sense of Peirce, as *iconic signs* for the presence or the absence of an outranking situation between the corresponding pair of decision actions.

Definition 10. A finite set $R = \{R_i \in \mathcal{R}_F \;/\; i = 1..m\}$ of m semiotical references, such that $R_i \cap R_j$ gives an empty reference $\forall i \neq j = 1..m$, is called a *mutually exclusive reference class*.

If these references significantly cover the whole consequence spectra $\Gamma(A)$, i.e $\bigcup_{i=1}^{m} R_i$ gives an exhaustive reference, we call this class an *exhaustive* one. A mutually exclusive and exhaustive reference class is also called a *complete semiotical reference system.*

We are now prepared for introducing the measure of the significance, i.e. the amount of truth assessment knowledge carried by each possible semiotical reference.

Definition 11. Let p denote an affirmative outranking assertion associated with a given reference family $\mathcal{R}_F$.

$\omega : \mathcal{R}_F \rightarrow \mathbb{Q}$ measures the *amount of truth assessment knowledge* captured by each possible semiotical reference concerning the potential truth of assertion p as a rational number verifying following structural conditions:

1 $\forall R \in \mathcal{R}_F \;:\; \omega(R) \geq 0$,

2 If $R = \{R_i \in \mathcal{R}_F \;/\; i = 1 \ldots m\}$ constitutes a mutually exclusive reference class then $\omega(R) = \sum_{i=1}^{m} \omega(R_i)$.

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[4]Generally speaking, semiotics only apply to social interpretations of signs. But in accordance with a Peircian approach, we may very well specialize semiotical considerations to local cognitive contexts, here the social working context of the decision maker. In this way, we explicitly introduce a social dimension into the decision aid model.

For any semiotical reference, the ω measure may be interpreted as its absolute *significance* measure, not in the sense of the *utility* of the pragmatic consequences referenced per se, as it is the case for instance in classic utility theory, but instead of the *amount of truth assessment knowledge* the preferential argument, modelled by the criterion-functions, is providing. The relative version of this significance measure is defined as follows.

Definition 12. Let p denote an affirmative assertion associated with a reference family $\mathcal{R}$ containing an exhaustive and mutually exclusive reference class U of strictly positive universal measure $\omega(U)$. We denote $k : \mathcal{R} \to \mathbb{Q}$, the relative version of the ω measure:

$$\forall R \in \mathcal{R}_F : k(R) = \frac{\omega(R)}{\omega(U)}.$$

$k(R)$ represents the *relative significance* that reference R takes in the truth assessment of assertion p. Thus k models a *weight distribution* on all possible partial arguments concerning the truth assessment of assertions p.

Proposition 2. *The measure k on $\mathcal{R}_F$ verifies the following conditions:*

1 $\forall R \in \mathcal{R}_F : k(R) \geq 0$.

2 *the weight of any exhaustive reference U equals* 1.

3 *the weight of an empty reference equals* 0.

4 *If $R = \{R_1, ..., R_m\}$ constitutes a mutually exclusive reference class then* $k(R) = \sum_{i=1}^{m} k(R_i)$.

Proof. All these conditions trivially follow from the defining properties (Definition 11) and the normalization (Definition 12) of the ω measure. □

This last proposition gives us the necessary elements for reformulating the concordance principle in terms of semiotical references.

3.3. Reformulating the concordance principle

Let p represent an affirmative assertion associated with a reference family $\mathcal{R}_F$. We denote $p_{|R}$ the *semiotical restriction* of assertion p to a given reference $R \in \mathcal{R}_F$.

The semiotical restriction principle generates for any affirmative assertion p, a family of *partial* assertions $p_{|R}$, one associated with each possible semiotical reference $R \in \mathcal{R}_F$.

Let us first concentrate on the truth assessment of such partial assertions, that are restricted to elementary references.

Definition 13. Let p represent an affirmative assertion associated with a reference family $\mathcal{R}_F$ containing a set of elementary references.

If $R_e \in \mathcal{R}$ represents such an elementary reference, the *degree of credibility* $r''(p_{|R_e})$ of assertion $p_{|R_e}$ is defined as follows:

$$r''(p_{|R_e}) = \begin{cases} 1 & \text{if } R_e \text{ certainly confirms assertion } p_{|R_e} \\ 0 & \text{otherwise} \end{cases}$$

Assertion $p_{|R_e}$ is warranted to be:

$$\begin{array}{lll} \textit{true} & \text{if} & r''(p_{|R_e}) = 1, \\ \textit{false} & \text{if} & r''(p_{|R_e}) = 0. \end{array}$$

Indeed, restricted to elementary preference dimensions, the constructive methodology allows us to assume that the relative measure of significance of the argument restricted to an elementary reference, is 1, in the sense that it is precisely the operational purpose of a criterion-function to *most clearly signify*, under the principle "toute chose pareille par ailleurs", what of the two possible truth values (*true* or *false*) is actually the casewhen looking at a given outranking situation.

Based upon these elementary references, we may now recursively define the degree of credibility of the universal assertion.

Definition 14. Let p represent an affirmative outranking assertion associated with a reference family $\mathcal{R}_F$ supporting a weight distribution k and let $\{R_i \in \mathcal{R}_F \;/\; i = 1 \ldots n\}$ denote a complete semiotical reference system. The degree r'' of credibility of assertion p is given by the following recursive definition:

$$r''(p) = \sum_{i=1}^{n} \big(k(R_i) \times r''(p_{|R_i})\big).$$

Assertion p is warranted to be:

$$\begin{array}{lll} \textit{more or less true} & \text{if} & r''(p) > \frac{1}{2}, \\ \textit{more or less false} & \text{if} & r''(p) < \frac{1}{2}, \\ \textit{logically undetermined} & \text{if} & r''(p) = \frac{1}{2}. \end{array}$$

Proposition 3. *Definitions 13 and 14 above are identical to the classic definition of the concordance principle (see Definition 5).*

Proof. Indeed, Definition 13 implements the split of the family of criteria into a subset F^+ of criteria in favour of the universal assertion, and the complementary subset F^- of criteria in disfavour. Definition 14 implements the balancing reasons proceeding, which we saw being isomorphic to the classic concordance principle (see Proposition 1). □

The elementary reference associated with each individual criterion $g \in F$ allows a clear partial truth assessment. In case of mutual exclusiveness and universal closure of the elementary references, universal outranking assertions may be truth assessed through a weighted mean of credibilities associated with the involved elementary consequences.

We may thus reformulate the coherence properties of the underlying family of criteria.

3.4. Reformulating the coherence axioms of the family of criteria

Aggregating the credibility degree for assertions with a composed reference, requires decomposing this reference into an exhaustive class of mutually exclusive elementary references.

Proposition 4. *Let A be a set of decision actions evaluated on a family F of criteria. F is* coherent *(in the sense of Definition 6) only if it provides each affirmative outranking assertion on A with a semiotical reference family containing a set of elementary references which constitutes a complete system.*

Proof. Indeed, Roy's coherence properties, i.e. exhaustiveness, cohesiveness ad minimality are all three implied by the fact that the elementary references associated with each individual criterion-function constitute an exhaustive and mutually exclusive reference class. □

It is worthwhile noticing that Proposition 4 shows a single implication from the conditions imposed on the semiotical reference family towards Roy's coherence properties of the family of criteria. The semiotical conditions appear as antecedent conditions for a possible coherence of the family of criteria whereas the latter formulate consequent conditions that constrain, mainly via the cohesiveness axiom, the out coming universal outranking relation.

We illustrate the semiotical foundation of the concordance principle with the following didactic example[5].

3.5. Practical example: Ranking statistic students

Three students in a Mathematics Department, specializing in statistics and denoted $\{a, b, c\}$, are to be ranked with respect to their competencies in the following subjects: *linear algebra* (la), *calculus* (ca) and *statistics* (st). The performances of the students in these three subjects are shown in Table 1.1. We suppose that in the eye of the assessor, each subject appears as an elementary reference for assessing the truth of his ranking assertions. We may notice here

[5] taken from Marichal [14, p. 192].

Table 1.1. Students performance tableau

student	la	ca	st
a	12	12	19
b	16	16	15
c	19	19	12

Table 1.2. Credibility of the pairwise outranking assertions

$r(x \, \mathrm{S} \, y)$	a	b	c
a	1.0	0,42	0,42
b	0,58	1,0	0,42
c	0,58	0,58	1,0

that the performance tableau shows in fact two opposite rankings, one common for the two pure mathematical subjects and one for statistics. Well, the two math results clearly support the ranking $a < b < c$ whereas the results in statistics support the opposite ranking: $a > b > c$. Let us furthermore suppose that the assessor admits the following significance weights for these elementary references in the truth assessment of his(her) global ranking: $\omega_{la} = 0.29$, $\omega_{ca} = 0.29$ and $\omega_{st} = 0.42$.

Under the hypothesis that the three elementary references constitute a complete semiotical reference system, we are indeed in presence of a coherent family of criteria and we may compute the credibilities of the pairwise outrankings shown in Table 1.2.

This valued global outranking relation clearly denotes the ranking supported by the pure math subjects, a result that may for instance not really convince the given assessor, a professor in statistics for instance. Indeed, (s)he would perhaps more expect the best student in statistics to come first. In this hypothetical case, the family of criteria would not verify one or the other of the three coherence requirements, i.e. exhaustiveness, cohesiveness and minimality. Following Proposition 4, we know now that an incoherent family of criteria implies in fact that the criteria don't provide in this case a complete semiotical reference system.

And indeed, let's suppose for instance that both statistics and calculus subjects present some *overlapping* with respect to their respective significance! Indeed, *calculus* and *statistics* subjects are typically not mutually exclusive with respect to their semantic content, at least in a Mathematics Department. A student who gets very high marks in *statistics* and relatively low ones in *calculus* presents therefore a somehow ambiguous profile. The statistician would tend to extend the high marks in *statistics* to the universal evaluation, whereas a pure mathematician would rather have the reflex to extend the low mark in *calculus* and *linear algebra* to his(her) universal evaluation.

We investigate such typical cases of incoherences in the next Section and show possible extensions to the concordance principle.

4. Extensions of the concordance principle

From the closing example of the previous section, we recognize that possible origins for incoherences in the family of criteria may be the following:

- *Overlapping criteria*: some elementary semiotical references are actually not mutually exclusive, i.e. the corresponding criteria appear to be *partly redundant*;

- *Incomplete performance tableau*: the set of elementary references supported by the criterion-functions don't provide an exhaustive reference class and/or we observe missing performances on some criteria.

Besides these inconsistencies above, one may naturally question the precise numerical measurability of the relative significance of each criteria. This is a well known weak point of the logical approach for multicriteria reference aggregation and many work-arounds have been proposed (see Mousseau [15]). We do not have the space here to discuss this issue, therefore we postpone this topic to a future publication[6] and concentrate now our attention first, on a situation where we may observe partly redundant criteria.

4.1. Pairwise redundant criteria

To illustrate the problem, we reconsider the evaluation of the statistics students. Let's assume that the assessor admits for instance some 50% overlap between the *statistics* and *calculus* subjects, i.e. 50% of the truth assessment knowledge involved in the *statistic* performances is also covered by the *calculus* performances. Numerically expressed, $50\% \times 0.42 = 0.21$ of the weight of *statistics*, or the other side round, $84\% \times 0.29 = 0.21$ of the weight of *calculus* is in fact shared by both arguments. More anchored in statistics for instance, our assessor exhibits, for judging this overlapping part, a tendency in favour of the very positive outcome of the statistics test. Whereas a more pure mathematics oriented assessor would consider first the less brilliant result of the same overlapping part in the context of the *calculus* test, thereby motivating his(her) more sceptical appreciation of student a. Overlapping of

[6] We have presented a purely ordinal version of the concordance principle in a communication at the 22nd Linz Seminar on Fuzzy Set Theory on *Valued Relations and Capacities in Decision Theory*, organised by E.P. Klement and M. Roubens, February 2001 (see Bisdorff [7]).

elementary references seems therefore introducing unstable and conflictuous relative weights of semiotical references.

We now formally introduce this potential overlapping of semiotical references.

Definition 15. Let p represent an affirmative assertion associated with a reference family $\mathcal{R}_F$. Let R_i and R_j be any two references from $\mathcal{R}_F$. We denote $R_{ij} = R_i \cap R_j$ the semiotical reference shared between R_i and R_j.

Theoretically, any possible figure of overlapping criteria may be described by the preceding formalism, but in practice we are only interested in partly pairwise overlapping criteria.

Definition 16. If no semiotical reference may be *shared by more than two* elementary references, i.e. overlapping between elementary references is reduced to pairs of elementary references, we say that the family of criteria is *pairwise decomposable.*

In a pairwise decomposable family of criteria, elementary references may be split into pairs of mutually exclusive elementary references. Adding to these shared references the exclusive part of each elementary reference, we obtain again a complete semiotical reference system, i.e. a mutually exclusive and exhaustive reference class.

Proposition 5. *Let $\mathcal{R}_F$ be the reference family associated with a pairwise decomposable exhaustive family of criteria F. Let R_{ii}, $\forall i = 1 \dots n$, represent the exclusive parts of each elementary reference $R_i \in \mathcal{R}_F$, i.e. $R_{ii} = R_i - \big(\cup_{i \neq j=1}^{n} R_{ij}\big)$. Then the pairwise decomposed reference class $R^2 = \{R_{ij} : i, j = 1 \dots n\}$ renders a complete semiotical reference system.*

Proof. Indeed, R^2 constitutes a partition covering completely all given elementary references:

$$\bigcup_{i=1}^{n}\bigcup_{j=1}^{n} R_{ij} = \bigcup_{i=1}^{n} R_i, \tag{1.2}$$

$$\forall i \neq j \ , \ R_{ii} \cap R_{jj} = \emptyset, \tag{1.3}$$

$$\forall (i,j) \neq (k,l) \ , \ R_{ij} \cap R_{kl} = \emptyset. \tag{1.4}$$

□

We may now evaluate the pairwise decomposed weight distribution supported by the new complete reference system R^2.

Definition 17. Let R_i and R_j be two different elementary references from $\mathcal{R}_F$ supporting respectively $\omega(R_i)$ and $\omega(R_j)$ amount of truth assessment knowledge concerning assertion p. The conditional weight coefficient

$$k_{j|i} = \frac{\omega(R_{ij})}{\omega(R_i)}$$

captures formally the overlapping of reference R_j with respect to reference R_i.

Knowing thus the overlapping part between two elementary references, we are able to compute the amount of truth assessment knowledge shared between them. It is important to notice, following Proposition 5, that such a decomposition of the elementary semiotical references returns in fact an exhaustive and mutually exclusive reference class. Therefore we are able to compute a relative weight distribution on the pairwise decomposed elementary references.

Definition 18. Let F be a pairwise decomposable exhaustive family of criteria and let R^2 (see Proposition 5) represent the set of pairwise decomposed elementary references. We denote $k_{ij}, i < j$ the relative weight associated with a shared semiotical reference R_{ij} and $k_{ii}, i = 1..n$ the relative weight associated with R_{ii}, the exclusive part of each elementary reference R_i. Let $\omega(R^2)$ represent the global amount of truth assessment knowledge supported by the complete system R^2. Formally, $\forall i, j = 1..n$ and $i \leq j$:

$$k_{ii} = \frac{\omega_i - \left(\sum_{k \neq i=1}^{n} \omega(R_{ik})\right)}{\omega(R^2)} \tag{1.5}$$

$$k_{ij} = \frac{\omega(R_{ij})}{\omega(R^2)} \tag{1.6}$$

In Table 1.3 we show the corresponding decomposition for the three subjects underlying the evaluation of the statistics students under the hypothesis that the *calculus* reference presents a 50% overlap with respect to *statistics* reference. The marginal distributions $k_{i.}$ and $k_{.j}$ shown in Table 1.3 allow two different

Table 1.3. Example of relative pairwise decomposed truth assessment weights

topics	la	ca	st	U
k_i	0,29	0,29	0,42	1,00
k_{ij}	la	ca	st	$k_{i.}$
la	0,37	0	0	*0,37*
ca	-	0,10	0,265	*0,365*
st	-	-	0,265	*0,265*
$k_{.j}$	*0,37*	*0,10*	*0,53*	1.00

semiotical interpretations of the pairwise decomposition of the elementary references, – the first more *statistics* and – the second, more *general mathematics* oriented. A more *statistics* oriented assessor could on the one hand adopt the weights $k_{.la} = 0.37$, $k_{.ca} = 0.10$ and $k_{.st} = 0.53$, with the consequence that the *statistics* results would prevail in the global ranking. The *math* oriented assessor on the other hand, could adopt the other limit weights, i.e. $k_{la.} = 0.37$, $k_{ca.} = 0.365$ and $k_{st.} = 0.265$ and thereby even more stress the ranking shown by both the math subjects.

Possible ambiguous interpretations appear thus as a sure sign of partial redundancy between criteria. Well, in order to stay faithful with our decision aid methodology, we will promote, in the absence of other relevant information, a neutral interpretation, situated in the middle between both extreme ones. To do so, we first extend the credibility calculus to pairwise shared references.

Definition 19. Let p represent an outranking assertion associated with a reference family $\mathcal{R}$ containing a set of pairwise decomposable elementary references. If $R_{ij} \in \mathcal{R}$ represents a shared reference between elementary references R_i and R_j, the *degree of credibility* $r(p_{|R_{ij}})$ of assertion $p_{|R_{ij}}$ is given as follows:

$$r(p_{|R_{ij}}) = \frac{r(p_{|R_i}) + r(p_{|R_j})}{2}$$

If both elementary references give unanimous results, either zero or one, the resulting credibility will be the same as the credibilities of the underlying elementary references. If they disagree, the degree of credibility of their shared reference part will be put to $\frac{1}{2}$, i.e. the logically undetermined value.

Now we may reformulate the general definition of the concordance for an universal outranking assertion based on a pairwise decomposable family of criteria.

Definition 20. Let p represent an outranking assertion evaluated on a pairwise decomposable and exhaustive family of criteria F. The corresponding pairwise decomposed elementary references are associated with a relative weight distribution k_{ij}.

The credibility $r(p)$ of assertion p is computed as follows:

$$r(p) = \sum_i \big(k_{ii} \cdot r(p_{|R_i})\big) + \sum_{ij:i<j} \big(k_{ij} \cdot r(p_{|R_{ij}})\big)$$

On the exclusive parts of the elementary references R_i, we keep the standard two-valued credibility denotation, as introduced in Definition 13. On the shared references however, we take the mean of both elementary credibilities, as formulated in Definition 19.

Reconsidering the ranking of the statistics students, we may notice in Table 1.4, that the original global outranking shown by the *mathematical* tests has been slightly more stressed as was before (see Table 1.2). This result, perhaps

Table 1.4. Outranking index from pairwise decomposable family of criteria

$r(x \,\mathrm{S}\, y)$	a	b	c
a	1,0	0,4	0,4
b	0,6	1,0	0,4
c	0,6	0,6	1,0

deceiving the *statistics* oriented assessor, is nevertheless 'logical' as we consider that a large part of the *calculus* test is now *interpreted* in fact as a *statistical* test. Let us close this section by showing that our extension is a compatible extension of the classical concordance principle.

Proposition 6. *The extended concordance principle of Definitions 19 and 20 is identical to the classic concordance principle (see Definition 5) if no overlapping is observed, i.e. if* $R_i \cap R_j = \emptyset \;\forall i, j = 1..n$.

Proof. Indeed, in this case, the exclusive part R_{ii} becomes identical to the original elementary reference R_i and we recover completely Definitions 13 and 14 of the concordance principle. □

Finally, we may notice, that this extension of the concordance principle to pairwise decomposable families of criteria still requires an exhaustive performance tableau. There exist however quite commonly decisions problems, where not all decision actions have been evaluated on all criteria (see [4, 5]). The classical concordance principle does not admit such missing evaluations. We now present a compatible extension for handling such situations.

4.2. Incomplete performance tableaux

We have extensively published our approach to incomplete performance tableaux (see Bisdorff [5, 8]) so that we may only briefly sketch this topic in the sequel.

Our idea is that in the limit, if two decision actions $a, b \in A$ have not been both evaluated on a given criterion function $g \in A$, the credibility $r(a \,\mathrm{S}_g\, b)$ given to the outranking assertion '$a \,\mathrm{S}\, b$' must take the logically undetermined value $\frac{1}{2}$.

Now, the more a decision action is missing common evaluations with all the others, the more its universal outranking with respect to all the others, is tending towards a credibility of $\frac{1}{2}$. Formally, we adjust Definition 13, giving the degree of credibility of an outranking situation observed on a single criterion, as follows.

Definition 21. Let '$a \,\mathrm{S}\, b$' represent an outranking assertion evaluated on a performance tableau involving a family F of criteria. For all $g \in F$, the degree of credibility $r(a\,\mathrm{S}_g\,b)$ of the outranking situation restricted to the semiotical reference of criterion g is defined as follows:

$$r(a\,\mathrm{S}_g\,b) = \begin{cases} 1 & \text{if } (g(a), g(b)) \text{ confirms assertion '}a\,\mathrm{S}_g\,b\text{'}, \\ \frac{1}{2} & \text{if } g(a) \text{ and/or } g(b) \text{ is undefined}, \\ 0 & \text{otherwise.} \end{cases}$$

Assertion '$a\,\mathrm{S}_g\,b$' is warranted to be:

$$\begin{array}{lll} \textit{true} & \text{if} & r(a\,\mathrm{S}_g\,b) = 1, \\ \textit{undetermined} & \text{if} & r(a\,\mathrm{S}_g\,b) = \frac{1}{2}, \\ \textit{false} & \text{if} & r(a\,\mathrm{S}_g\,b) = 0. \end{array}$$

With this extension of the concordance principle, we weight the universal outranking index $r(a\,\mathrm{S}\,b)$ with the relative frequency of common evaluations, and we add halve of the relatively missing evaluations as confirming and the other halve as not confirming the given assertion.

This technique allows us to take into account at the same time sporadic missing evaluations, but also completely missing criteria. In the latter case, all decision actions will compare on a missing criterion with a credibility of $\frac{1}{2}$, i.e. the logical denotation of the outranking assertion, restricted to the missing criterion, will be undetermined for all couples of decision actions.

5. Conclusion

In this chapter, we have investigated the logical and semiotical foundation of the concordance principle, i.e. the logical approach to multicriteria preference aggregation promoted since 1970 by Bernard Roy.

We first showed the denotational isomorphism which exists between this concordance principle and the proceeding of balancing reasons as promoted by C.S. Peirce. This result illustrates the split truth versus falseness denotation installed by the concordance principle.

Taking furthermore support on the Peircian distinction between credibility and state of belief concerning a preferential assertion, we propose a semiotical foundation for the numerical determination of the significance, i.e. the truth assessment knowledge carried by the family of criteria. This approach makes apparent the semiotical requirements guaranteeing the coherence of a family of criteria.

Finally, based on these semiotical requirements, we propose an extension of the concordance principle in order to support pairwise overlapping criteria and/or incomplete performance tableaux.

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