RUBIS : a bipolar-valued outranking method for the choice method

Raymond Bisdorff

Applied Mathematics Unit, University of Luxembourg

2nd Decision Deck workshop, Paris, November 16, 2007

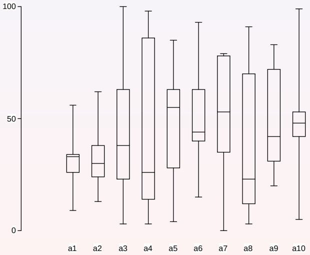
Introductory example

Decision:problem: *Choose the best from a set of ten alternatives evaluated on 7 criteria as shown below.*

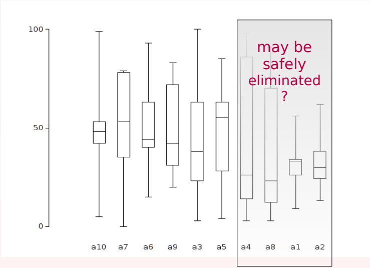
criterion	weight	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
g_1	7	33	13	3	14	48	44	18	47	31	98
g_2	7	9	30	23	86	63	40	79	3	83	48
g_3	5	34	38	63	16	85	53	78	91	47	42
g_4	5	53	24	38	3	28	93	35	12	72	5
g_5	5	26	44	60	98	62	15	53	23	37	44
g_6	4	26	29	100	36	4	63	54	70	24	53
g_7	1	56	62	33	36	21	49	0	13	20	99

- The performance scale on each criteria is $0 - 100$ pts, with a weak preference threshold of 10 points, a preference threshold of 20 pts, and a veto threshold of 80 pts.
- We assume that the criteria are not commensurable.

Introductory example: Boxplots of the performances



Introductory example: Boxplots of the performances



Introductory example: Ranking the performances?

Criterion	a10	a7	a6	a9	a3	a5	a4	a8	a1	a2
"g1"	98	18	44	31	3	48	14	47	33	13
"g2"	48	79	40	83	23	63	86	3	9	30
"g3"	42	78	53	47	63	85	16	91	34	38
"g4"	5	35	93	72	38	28	3	12	53	24
"g5"	44	53	15	37	60	62	98	23	26	44
"g6"	53	54	63	24	100	4	36	70	26	29
"g7"	99	0	49	20	33	21	36	13	56	62

Introductory example: Pairwise comparisons

Is a_{10} globally at least as good as a_7?

Outranking thresholds: weak preference ($\geqslant 10$), preference ($\geqslant 20$), veto ($\leqslant -80$).

criterion	w_i	a_{10}	a_7	$\Delta_i(10,7)$	balance	veto ?
g_1	7	98	18	80	+7	no
g_2	7	48	79	-31	-7	no
g_3	5	42	78	-36	-5	no
g_4	5	5	35	-30	-5	no
g_5	5	44	53	-9	+5	no
g_6	4	53	54	-1	+4	no
g_7	1	99	0	99	+1	no
				total balance	0	

We observe a balanced situation.
No conclusion can be drawn.

Introductory example: Pairwise comparisions (continued)

Is a_7 globally at least as good as a_{10}?

Outranking thresholds: weak preference ($\geqslant 10$), preference ($\geqslant 20$), veto ($\leqslant -80$).

criterion	w_i	a_7	a_{10}	$\Delta_i(10,7)$	balance	veto ?
g_1	7	18	98	-80	-7	yes
g_2	7	79	48	+31	+7	no
g_3	5	78	42	+36	+5	no
g_4	5	35	5	+30	+5	no
g_5	5	53	44	+9	+5	no
g_6	4	54	53	+1	+4	no
g_7	1	0	99	-99	-1	yes
				total balance	+18-34	

We observe a veto situation on criteria g_1 and g_7.
a_7 is clearly not globally at least as good as a_{10}? !

Introductory example: Pairwise comparisions (continued)

Is a_{10} (resp. a_6) globally at least as good as a_6 (resp. a_{10}) ?

g_i	w_i	a_{10}	a_6	$\Delta_i(10,6)$	balance	veto?	$\Delta_i(6,10)$	balance	veto?
g_1	7	98	44	54	+7	no	-54	-7	no
g_2	7	48	40	8	+7	no	-8	+7	no
g_3	5	42	53	-11	-5	no	11	+5	no
g_4	5	5	93	-88	-5	yes	88	+5	no
g_5	5	44	15	29	+5	no	-29	-5	no
g_6	4	53	63	-10	0	no	10	+4	no
g_7	1	99	49	50	+1	no	-50	-1	no
				total balance	~~+10~~ -34		total balance	+8	

- a_{10} is clearly not globally at least as good as a_6 (veto (-88) on criterion g_4)!
- Note the weak preference situation on criterion g_6 !
- a_6 is globally at least as good as a_{10} (balance of +8 in favour).

Introductory example: Pairwise comparisions (continued)

Is a_6 globally at least as good as a_7 ?

criteria	weight	a_7	a_6	$\Delta_i(7,6)$	balance	veto ?
g_1	7	44	18	26	+7	no
g_2	7	40	79	-39	-7	no
g_3	5	53	78	-25	-5	no
g_4	5	93	35	58	+5	no
g_5	5	15	53	-38	-5	no
g_6	4	63	54	9	+4	no
g_7	1	49	0	49	+1	no
				total balance	0	

We observe again a balanced situation.
No conclusion can be drawn.

Introductory example: Global outranking relation

$\tilde{S}$	a_{10}	a_7	a_6	a_9	a_3	a_5	a_4	a_8	a_1	a_2
a_{10}	-	0	-34	10	1	2	10	20	24	29
a_7	-34	-	8	15	24	18	22	10	20	32
a_6	8	0	-	10	11	0	-34	24	29	23
a_9	10	11	7	-	10	7	19	9	32	32
a_3	-34	8	2	-4	-	-4	3	10	13	25
a_5	10	19	14	2	-34	-	1	26	14	24
a_4	-34	10	-34	7	6	0	-	2	10	12
a_8	-34	0	-34	-34	-10	5	-34	-	22	3
a_1	-9	-8	-10	5	-1	-7	6	9	-	15
a_2	-34	-3	-10	3	6	-9	10	2	10	-

Backbone of Rubis : $\widetilde{S}$

- Let X be a finite set of p alternatives.
- Let N be a finite set of $n > 1$ criteria.
- Let m be the total significance of the criteria.
- Let x and y be two alternatives from X.
- Let x_i be the value taken by x on criterion g_i

Definition (The outranking situation)

- x outranks y ($x \, S \, y$) if there is a significant majority of criteria which support an at least as good statement and there is no criterion which raises a veto against it.
- The bipolar valued relation $\widetilde{S} \in [-m, m]$ expresses the credibility of the validation or the non-validation of the outranking relation S.

Backbone of RUBIS : $\widetilde{S}$

Definition (The bipolar valued outranking situation)

$$\widetilde{S}(x,y) = \min\Big\{ \big(\sum_{i \in N} w_i \cdot C_i(x,y) \big), \min_{i \in N} \big(-V_i(x,y) \big) \cdot m \Big\}$$

$$C_i(x,y) = \begin{cases} 1 & \text{if} \quad x_i + q_i > y_i; \\ -1 & \text{if} \quad x_i + p_i \leqslant y_i \\ 0 & \text{otherwise} \end{cases}$$

$$-V_i(x,y) = \begin{cases} 1 & \text{if} \quad x_i + wv_i > y_i; \\ -1 & \text{if} \quad x_i + v_i \leqslant y_i \\ 0 & \text{otherwise} \end{cases}$$

where q_i, p_i represent the weak preference, resp. the preference, and wv_i, v_i, the weak veto, resp. the veto, threshold on criterion g_i.

Backbone of RUBIS : $\widetilde{S}$

$\widetilde{S}$ is defined on a bipolar-valued credibility scale $\mathcal{L} = [-m, m]$ supporting the following demantics denotation:

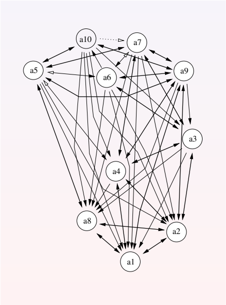
- $\widetilde{S}(x,y) = +m$ means that assertion $x \, S \, y$ is clearly validated.
- $\widetilde{S}(x,y) = -m$ means that assertion $x \, S \, y$ is clearly non-validated.
- $\widetilde{S}(x,y) > 0$ means that assertion $x \, S \, y$ is more validated than non-validated.
- $\widetilde{S}(x,y) < 0$ means that assertion $x \, S \, y$ is more non-validated than validated.
- $\widetilde{S}(x,y) = 0$ means that assertion $x \, S \, y$ is undetermined.

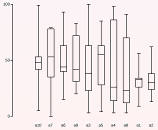
Backbone of RUBIS : $\widetilde{G}(X, \widetilde{S})$

Definition (The bipolar valued outranking digraph)

- We denote $\widetilde{G}(X, \widetilde{S})$ the bipolar-valued outranking digraph modelled via $\widetilde{S}$ on $X \times X$.
- The associated crisp outranking relation S may be recovered from $\widetilde{S}$ as the set of pairs (x,y) such that $\widetilde{S} > 0$.
- $G(X, S)$ is called the crisp outranking digraph associated with $\widetilde{G}(X, \widetilde{S})$.

Introductory example: The crisp outranking digraph





RUBIS decision aiding approach

- A choice problem traditionally consists in the search for a single best alternative.
- We adopt a progressive decision analysis process which allows to uncover the best single choice via possible intermediate recommendations.
- These intermediate choice recommendations, the case given, have to be refined at some further stages of the decision analysis.

Pragmatic choice recommendation (CR) principles

$\mathcal{P}_1$: Non-retainement for well motivated reasons.
all eliminated alternative must be considered worse as at least one recommended alternative.

$\mathcal{P}_2$: Minimal size.
the CR should be as limited as possible.

$\mathcal{P}_3$: Efficient and informative.
each CR must deliver a stable recommandation.

$\mathcal{P}_4$: Effectively better.
the CR should not correspond simultaneously to a choice and an elimination recommendation.

$\mathcal{P}_5$: Maximally credible.
the CR must be as credible as possible wrt the preferential knowedge modelled via $\widetilde{\mathrm{S}}$.

Useful choice qualifications in $\widetilde{G}(X, \widetilde{\mathrm{S}})$

Let Y be a non-empty subset of X, called a choice in $\widetilde{G}$.

- Y is said to be outranking (resp. outranked) iff $x \notin X \Rightarrow \exists y \in Y : \widetilde{\mathrm{S}}(x, y) > 0)$.
- Y is said to be independent iff for all $x \neq y$ in Y we have X $\widetilde{\mathrm{S}}(x, y) \leqslant 0)$.
- Y is called an outranking kernel (resp. outranked kernel) iff it is an outranking (resp. outranked) and indendent choice.
- Y is called an outranking hyperkernel (resp. outranked hyperkernel) iff it is an outranking (resp. outranked) choice which consists of independent chordless circuits of odd order $p \geqslant 1$.

Tranlating CR principles into choice qualifications

$\mathcal{P}_1$: Non-retainment for well motivated reasons.
A CR is an outranking choice.

$\mathcal{P}_{2+3}$: Minimal size & stable.
A CR is a hyperkernel.

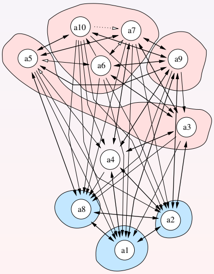
$\mathcal{P}_4$: Effectivity.
A CR is a stricly more outranking than outranked choice.

$\mathcal{P}_5$: Maximal credibility.
A CR has maximal determinateness.

Theorem

Any bipolar outranking digraph contains at least one outranking and one outranked hyperkernel.

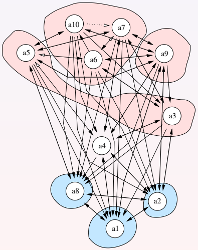
Introductory example: All outranking and outranked hyperkernels



The RUBIS choice recommendation (RCR)

- A RCR verifies the five CR principles.
- A maximally determined strict outranking hyperkernel, if it exists in $\widetilde{G}$, gives a RCR.
- A RCR is a provisional subset of alternatives, most certainly containing the best alternative, if it exists !.
- A RCR must not be confused with the ultimate best choice of the decision maker.
- The RUBIS choice method is only convenient in a progressive decision aiding approach.

Introductory example: all kernels and hyperkernels




```
outranking choices:
```
$\{a_{10}, a_7, a_6\}$

$\{a_9\}$

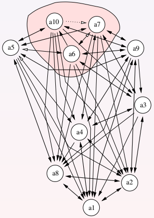
$\{a_3, a_5\}$

```
outranked choices:
```
$\{a_8\}$

$\{a_2\}$

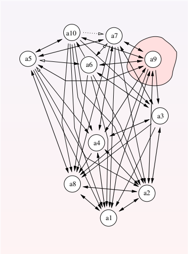
$\{a_1\}$

Introductory example: The RUBIS choice recommendation



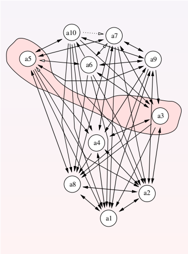
choice : $\{a_{10}, a_7, a_6\}$
(chordless 3-circuit!)
determinateness : 72%
(weighted majority of criterion)
irredundancy : 100%
independence : 100%
outrankingness : 72%
outrankedness : 38%
characteristic vector = [
$\{a_{10}, a_7, a_6\}$: 72%, a_1: 28%,
a_2: 28%, a_3: 28%, a_4: 28%,
a_5: 28%, a_6: 28%, a_7: 28%, a_8: 28%, a_9: 28%, a_{10}: 28%]

Introductory example: Potential choice recommendation



choice : $\{a_9\}$
determinateness : 60%
(weighted majority of criterion)
irredundancy : 100%
independence : 100%
outrankingness : 60%
outrankedness : 0%
characteristic vector = [a_9: 60%, a_1: 40%, a_2: 40%, a_3: 40%, a_4: 40%, a_5: 40%, a_6: 40%, a_7: 40%, a_8: 40%, a_{10}: 40%, $\{a_{10}, a_7, a_6\}$: 40%]

Introductory example: Other potential choice recommendation



choice : $\{a_3, a_5\}$
determinateness : 53%
(weighted majority of criterion)
irredundancy : 65%
independence : 56%
outrankingness : 53%
outrankedness : 48.5%
characteristic vector = [a_3: 53%, a_5: 53%, a_1: 47%, a_2: 47%, a_4: 47%, a_6: 47%, a_7: 47%, a_8: 47%, a_{10}: 47%, $\{a_{10}, a_7, a_6\}$: 47%]

Concluding remarks

Properties of the RUBIS choice recommendation:

- Progressiveness: intermediate solutions are proposed to the decision maker;
- Existence: A RCR always exists in a non-symmetrical bipolar-valued outranking digraph;
- Multiplicity: In case multiple RCR coexist, their union gives a suitable intermediate choice recommendation;
- Missing values: They are treated as information which is not available at a given stage of the decision analysis; which might be determined later on;
- Efficient decision aiding: Strongly motivated conclusions can nevertheless be drawn.

Bibliography I

R. Bisdorff, P. Meyer, M. Roubens (2007)
RUBIS: a bipolar-valued outranking method for the choice problem.
4OR, A Quarterly Journal of Operations Research, Springer-Verlag,forthcoming.
(Online) Electronic version: DOI: 10.1007/s10288-007-0045-5.

R. Bisdorff, M. Pirlot and M. Roubens (2006).
Choices and kernels from bipolar valued digraphs.
European Journal of Operational Research, 175 (2006) 155-170.

R. Bisdorff (2007)
The Python Digraph implementation for Ruby: User manual.
University of Luxembourg, 2007,
http://sma.uni.lu/bisdorff/Digraph

Bibliography II

D2-Decision-Deck Consortium (2007)
The D2-Decision-Deck project.
http://www.decision-deck.org/